Abstract—Power distribution network (PDN) of a multilayered PCB is designed to supply low noise and stable power to ICs. Reduced voltage levels, increased current requirements make it challenging to attain the desired PDN impedance profile. It is therefore necessary to have multiple design iterations for optimal performance of the PDN. 3D full-wave electromagnetic solvers like the Partial Element Equivalent Circuit (PEEC) method are time constrained and therefore ill-suited for early stage design. On the other hand, 2.5D tools have lower time and memory requirements and are reasonably accurate for planar power-ground structures. For example, Multilayered Finite Difference Method (MFDM) is a 2.5D formulation suitable for PDN analysis. However, present MFDM techniques are based on orthogonal meshes, such that power-ground planes with irregular shapes and holes require unnecessarily fine mesh at the boundary for a suitable staircase approximation. In this paper, a non-orthogonal 2.5D PEEC formulation is proposed to alleviate this problem. Numerical results using quadrilateral meshes demonstrate good accuracy as compared to 3D full-wave formulation for planar geometries.

Keywords—Power delivery network (PDN), Partial Element Equivalent Circuit (PEEC)

I. INTRODUCTION

With reducing voltage levels and increased current requirements, power integrity (PI) analysis become a critical component of chip-package-system design. Noise on power supply manifests itself as rail collapse and switching noise, with direct impact on noise margins and timing. The performance of a PDN is typically analyzed in the frequency domain. The target impedance of a PDN is a common criterion for practical engineering designs to ensure a voltage fluctuation smaller than the tolerable limit. PI analysis for PDN become challenging with multicore chips, mixed-signal designs with multiple references, and complex packaging situations such as system-in-package (SiP), system-on-chip (SoC), stacked die, package-on-package (PoP), and 3D ICs. Efficient PI simulation tools are required for accurate analysis of the PDN design through multiple design iterations such that optimal performance can be achieved. 3D electromagnetic full wave solvers like PEEC [1-2] for PDN analysis are limited by time and memory bottlenecks that make them unsuitable in an iterative design process.

In the past, several methods have been proposed for PDN analysis e.g. Transmission Matrix Method [3], cavity resonance method [4], Finite Difference Frequency Domain (FDFD) technique [5]. Multilayered finite-difference method [6-7] and Finite Element Method (FEM) for small feature size [8]. Existing MFDM combines FDFD and transmission line method to generate an equivalent circuit model for PDNs. In [6], a uniform square grid is used for the analysis, but the resultant uniform meshing leads to efficiency bottlenecks. In [7] a non-uniform non-conformal rectangular grid based method is proposed to preclude the requirement of uniform meshing. However, with orthogonal discretization, curved surfaces need staircase approximation which leads to inaccuracies if the geometry approximation is inadequate and time-inefficiency due to too many mesh elements for an appropriate staircase model.

In this paper, a non-orthogonal 2.5D PEEC formulation is proposed to cater to the requirements of modern power-ground planes with Swiss cheese patterns. A generalized formulation has been derived for quadrilateral cell parameters. The rest of the paper is organized as follows: section II briefly discusses the adopted non-orthogonal coordinate-system. The non-orthogonal 2.5D PEEC formulation is presented in section III followed by numerical results in section IV.

II. NON-ORTHOGONAL COORDINATES

In this section, a brief overview of the planar non-orthogonal co-ordinate system is given. Details of non-orthogonal system are described in several texts [2, 9]. Local coordinates $a$ and $b$ are used to separately represent each quadrilateral mesh-element.

![Fig. 1. Basic Quadrilateral element with local coordinate](image)

Four mapping functions are used to uniquely map each point $(a, b)$ into global Cartesian system $(x, y)$.

\[ X = N1(a,b) \times x1 + N2(a,b) \times x2 + N3(a,b) \times x3 + N4(a,b) \times x4 \]  

(1)
The mapping functions are defined as:
\[ N1 = (1/4) \times (1-a)(1-b) \]
\[ N2 = (1/4) \times (1+a)(1-b) \]
\[ N3 = (1/4) \times (1-a)(1+b) \]
\[ N4 = (1/4) \times (1+a)(1+b) \]

The directional derivative of position vector \((r_g)\) from origin with respect to local coordinate system can be defined as:
\[ \frac{\partial r_g}{\partial a} = \frac{\partial x}{\partial a} + \frac{\partial y}{\partial a}, \frac{\partial r_g}{\partial b} = \frac{\partial x}{\partial b} + \frac{\partial y}{\partial b} \]

The magnitude of the directional derivative is defined as \(h_a\) and \(h_b\).

**III. NON-ORTHOGONAL 2.5D PEEC FORMULATION**

A unit non-orthogonal 2.5D PEEC cell can be defined as in Fig. 2, where both top and bottom planes have the same shape.

**a) Resistance of unit cell:**

The resistance of a quadrilateral shape can be found as described in non-orthogonal PEEC [7]. For loop-resistance in non-orthogonal PEEC, the resistance in \(a\) and \(b\) direction can be computed as:
\[ R_a = 2 \int_{-1}^{1} \int_{-1}^{1} \frac{\rho}{\mu} \left(\frac{\partial r_g}{\partial a} \times \frac{\partial r_g}{\partial b} \right)^2 \, dadb \]

\[ R_b = 2 \int_{-1}^{1} \int_{-1}^{1} \frac{\rho}{\mu} \left(\frac{\partial r_g}{\partial b} \times \frac{\partial r_g}{\partial a} \right)^2 \, dadb \]

The factor 2 comes from the fact that the loop resistance includes conductor both in top and bottom planes.

**b) Loop-Inductance of unit cell**

Magnetic flux is perpendicular to current direction as shown in Fig. 3. For an area having unit length in \(a\) and \(b\) direction magnetic flux can be computed as described below:

![Fig. 3. Inductance of area of unit length in a and b](image)

From Ampere’s law, magnetic flux density for current \(I_a\) in \(a\) direction can be found as:
\[ B = \mu I_a \int \left(\frac{\partial r_g}{\partial b} \right) \sin \theta \, db \]

The denominator can also be written as:
\[ \left(\frac{\partial r_g}{\partial b} \right) \sin \theta = (\frac{\partial r_g}{\partial b} \times (\hat{a} \times \hat{z})) \]

Total loop inductance of the quadrilateral can be found by integrating over the area.

\[ L_a = \int \int \frac{\mu I_a}{\left(\frac{\partial r_g}{\partial b} \times \frac{\partial r_g}{\partial a} \right)} \, dadb \]
Similarly, inductance in $b$ direction can be derived as:

$$L_b = \int \frac{\mu d \frac{\partial r_z}{\partial b}^2}{\int_{-1}^{1} \left( \frac{\partial r_z}{\partial a} \right) \left( \frac{\partial r_z}{\partial b} \right) \cdot 2} \, dadb$$  \hspace{1cm} (12)

c) Capacitance and conductance of unit cell:

Since Electric Field is considered uniform between the plates and directed along the height, capacitance can be defined as

$$C = \varepsilon_{r} \varepsilon_{0} \frac{A}{d}$$  \hspace{1cm} (13)

where $\varepsilon_{r}$ is relative permittivity of the substrate and $A$ is area of the quadrilateral. Similarly, conductance ($G$) can be computed as:

$$G = \frac{\omega C \tan \delta}{\tan \delta}$$  \hspace{1cm} (14)

where $\tan \delta$ is the loss tangent of the dielectric material.

(d) 2.5D PEEC SPICE model:

Each cell parameters ($R,L,G,C$) are computed and assembled to form a netlist for frequency domain spice simulation as shown in Fig. 4.

Fig. 4. Spice model creation from 2.5D PEEC cells

IV. NUMERICAL RESULTS

To check the versatility of the proposed method various structures are analyzed. Gmsh is used to generate conformal quadrilateral meshes. In the first case, two power planes separated by 0.12mm FR4 dielectric is analyzed both using a 3D full wave commercial simulator and the proposed non-orthogonal 2.5D PEEC algorithm. The structure is similar to the one used in [7], but with some non-orthogonality introduced. Details of the geometry are described in Fig. 5 and the mesh is shown in Fig. 6.

Fig. 5. Geometry of power plane (dimension in mm)

Fig. 6. Quadrilateral mesh Power plane

The input impedance is calculated at port1 and compared with the results using a full wave simulator as shown in Fig. 7.

Fig. 7. Comparison of Input impedance
In the second case, a three layered power plane structure of size 44mm*50mm with a hole in middle plane as in Fig. 8 is considered. The structure is similar to the one in [6] with a circular hole instead of rectangular.

Fig. 8. Three layered Power plane

The structure is meshed using quadrilateral elements as shown in Fig. 9. The transfer coefficient between the ports is compared with that obtained from a 3D full wave commercial solver. It can be observed that coupling between the ports is maximum at cavity modes of the plane and it is effectively captured by our formulation.

Fig. 9. Quadrilateral Mesh Power plane with aperture

Fig. 10. Magnitude of transmission coefficient S21

V. CONCLUSION

A 2.5D non-orthogonal PEEC formulation is presented in this paper for power-ground geometries. Such a formulation can be used as a generalized approach to enhance the speed, and accuracy of 2.5D PDN analysis for realistic package-board PDN layout. Results demonstrate good agreement with 3D full-wave commercial tool results.

REFERENCES