ECE 497 JS Lecture -07
Planar Transmission Lines

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Microstrip

\[ Z_o = \frac{119.9}{\sqrt{2(\varepsilon_r + 1)}} \ln \left[ 4 \frac{h}{w} + \sqrt{16 \left( \frac{h}{w} \right)^2 + 2} \right] \]

for \( w/h < 3.3 \)

\[ Z_o = \frac{119.9\pi}{2\sqrt{\varepsilon_r}} \left\{ \frac{w}{2h} + \frac{\ln(4)}{\pi} + \frac{\ln(e\pi^2/16)}{2\pi} \left( \frac{\varepsilon_r - 1}{\varepsilon_r^2} \right) \right. \]
\[ \left. + \frac{\varepsilon_r + 1}{2\pi\varepsilon_r} \left[ \frac{\pi e}{2} + \ln \left( \frac{w}{2h} + 0.94 \right) \right] \right\} \]

for \( w/h > 3.3 \)
Three-Line Microstrip

\[- \frac{\partial V_1}{\partial z} = L_{11} \frac{\partial I_1}{\partial t} + L_{12} \frac{\partial I_2}{\partial t} + L_{13} \frac{\partial I_3}{\partial t} \]

\[- \frac{\partial V_2}{\partial z} = L_{21} \frac{\partial I_1}{\partial t} + L_{22} \frac{\partial I_2}{\partial t} + L_{23} \frac{\partial I_3}{\partial t} \]

\[- \frac{\partial V_3}{\partial z} = L_{31} \frac{\partial I_1}{\partial t} + L_{32} \frac{\partial I_2}{\partial t} + L_{33} \frac{\partial I_3}{\partial t} \]

\[- \frac{\partial I_1}{\partial z} = C_{11} \frac{\partial V_1}{\partial t} + C_{12} \frac{\partial V_2}{\partial t} + C_{13} \frac{\partial V_3}{\partial t} \]

\[- \frac{\partial I_2}{\partial z} = C_{21} \frac{\partial V_1}{\partial t} + C_{22} \frac{\partial V_2}{\partial t} + C_{23} \frac{\partial V_3}{\partial t} \]

\[- \frac{\partial I_3}{\partial z} = C_{31} \frac{\partial V_1}{\partial t} + C_{32} \frac{\partial V_2}{\partial t} + C_{33} \frac{\partial V_3}{\partial t} \]
Subtract (1c) from (1a) and (2c) from (2a), we get
\[-\frac{\partial V_\alpha}{\partial z} = (L_{11} - L_{13}) \frac{\partial I_\alpha}{\partial t} - \frac{\partial I_\alpha}{\partial z} = (C_{11} - C_{13}) \frac{\partial V_\alpha}{\partial t}\]

This defines the Alpha mode with:

\[V_\alpha = V_1 - V_3 \quad \text{and} \quad I_\alpha = I_1 - I_3\]

The wave impedance of that mode is:

\[Z_\alpha = \sqrt{\frac{L_{11} - L_{13}}{C_{11} - C_{13}}}\]

and the velocity is

\[u_\alpha = \frac{1}{\sqrt{(L_{11} - L_{13})(C_{11} - C_{13})}}\]
Three-Line – Modal Decomposition

In order to determine the next mode, assume that

\[ V_{\beta} = V_1 + \beta V_2 + V_3 \]
\[ I_{\beta} = I_1 + \beta I_2 + I_3 \]

\[ -\frac{\partial V_{\beta}}{\partial z} = (L_{11} + \beta L_{21} + L_{31}) \frac{\partial I_1}{\partial t} + (L_{12} + \beta L_{22} + L_{32}) \frac{\partial I_2}{\partial t} + (L_{13} + \beta L_{23} + L_{33}) \frac{\partial I_3}{\partial t} \]

\[ -\frac{\partial I_{\beta}}{\partial z} = (C_{11} + \beta C_{21} + C_{31}) \frac{\partial V_1}{\partial t} + (C_{12} + \beta C_{22} + C_{32}) \frac{\partial V_2}{\partial t} + (C_{13} + \beta C_{23} + C_{33}) \frac{\partial V_3}{\partial t} \]

By reciprocity \( L_{32} = L_{23}, L_{21} = L_{12}, L_{13} = L_{31} \)

By symmetry, \( L_{12} = L_{23} \)

Also by approximation, \( L_{22} \approx L_{11}, L_{11} + L_{13} \approx L_{11} \)
Three-Line – Modal Decomposition

\[-\frac{\partial V_\beta}{\partial z} = (L_{11} + \beta L_{12} + L_{13}) \left( \frac{\partial I_1}{\partial t} + \frac{\partial I_3}{\partial t} \right) + (2L_{12} + \beta L_{11}) \frac{\partial I_2}{\partial t} \]

In order to balance the right-hand side into \( I_\beta \), we need to have

\[ (2L_{12} + \beta L_{11}) I_2 = \beta (L_{11} + \beta L_{12} + L_{13}) I_2 = \beta (L_{11} + \beta L_{12}) I_2 \approx 2L_{12} = \beta^2 L_{12} \]

or \( \beta = \pm \sqrt{2} \)

Therefore the other two modes are defined as

The Beta mode with
Three-Line – Beta Mode

The Beta mode with

\[ V_\beta = V_1 + \sqrt{2}V_2 + V_3 \]

\[ I_\beta = I_1 + \sqrt{2}I_2 + I_3 \]

The characteristic impedance of the Beta mode is:

\[ Z_\beta = \frac{L_{11} + \sqrt{2}L_{12} + L_{13}}{\sqrt{C_{11} + \sqrt{2}C_{12} + C_{13}}} \]

and propagation velocity of the Beta mode is

\[ u_\beta = \frac{1}{\sqrt{(L_{11} + \sqrt{2}L_{12} + L_{13})(C_{11} + \sqrt{2}C_{12} + C_{13})}} \]
Three-Line – Delta Mode

The Delta mode is defined such that

\[ V_\delta = V_1 - \sqrt{2}V_2 + V_3 \]

\[ I_\delta = I_1 - \sqrt{2}I_2 + I_3 \]

The characteristic impedance of the Delta mode is

\[ Z_\delta = \sqrt{\frac{L_{11} - \sqrt{2}L_{12} + L_{13}}{C_{11} - \sqrt{2}C_{12} + C_{13}}} \]

The propagation velocity of the Delta mode is:

\[ u_\delta = \frac{1}{\sqrt{\left( L_{11} - \sqrt{2}L_{12} + L_{13} \right) \left( C_{11} - \sqrt{2}C_{12} + C_{13} \right)}} \]
Symmetric 3-Line Microstrip

In summary: we have 3 modes for the 3-line system

\[ E = \begin{bmatrix} 1 & 0 & -1 \\ 1 & \sqrt{2} & 1 \\ 1 & -\sqrt{2} & 1 \end{bmatrix} \]

Alpha mode  
Beta mode*  
Delta mode*

*neglecting coupling between nonadjacent lines
Coplanar Waveguide

\[ \varepsilon_r = 4.3 \]

\[
L(nH/m) = \begin{pmatrix}
346 & 162 & 67 \\
152 & 683 & 152 \\
67 & 162 & 346 \\
\end{pmatrix} \quad C(pF/m) = \begin{pmatrix}
113 & 17 & 5 \\
16 & 53 & 16 \\
5 & 17 & 113 \\
\end{pmatrix}
\]

\[
E = \begin{pmatrix}
0.45 & 0.12 & 0.45 \\
0.5 & 0 & -0.5 \\
-0.45 & 0.87 & -0.45 \\
\end{pmatrix} \quad H = \begin{pmatrix}
0.44 & 0.49 & 0.44 \\
0.5 & 0 & -0.5 \\
-0.10 & 0.88 & -0.10 \\
\end{pmatrix}
\]
Coplanar Waveguide

\[ Z_m(\Omega) = \begin{pmatrix} 73 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 94 \end{pmatrix} \quad Z_c(\Omega) = \begin{pmatrix} 56 & 23 & 8 \\ 22 & 119 & 22 \\ 8 & 23 & 56 \end{pmatrix} \]

\[ v_p(m/ns) = \begin{pmatrix} 0.15 & 0 & 0 \\ 0 & 0.17 & 0 \\ 0 & 0 & 0.18 \end{pmatrix} \]

\[ \varepsilon_r = 4.3 \]
Coplanar Waveguide

\[ K(k) : \text{Complete Elliptic Integral of the first kind} \]

\[ k = \frac{S}{S + 2W} \]

\[ K'(k) = K(k') \]

\[ k' = (1 - k^2)^{1/2} \]

\[ Z_{ocp} = \frac{30\pi}{\sqrt{\varepsilon_r + 1}} \frac{K'(k)}{K(k)} \text{ (ohm)} \]

\[ v_{cp} = \left( \frac{2}{\varepsilon_r + 1} \right)^{1/2} c \]
Coplanar Strips

\[ Z_{ocs} = \frac{120\pi}{\varepsilon_r + 1} \frac{K'(k)}{\sqrt{\frac{K(k)}{2}}} \text{ (ohm)} \]
Qualitative Comparison

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Microstrip</th>
<th>Coplanar Wguide</th>
<th>Coplanar strips</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\text{eff}}^*$</td>
<td>~6.5</td>
<td>~5</td>
<td>~5</td>
</tr>
<tr>
<td>Power handling</td>
<td>High</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Radiation loss</td>
<td>Low</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Unloaded Q</td>
<td>High</td>
<td>Medium</td>
<td>Low or High</td>
</tr>
<tr>
<td>Dispersion</td>
<td>Small</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Mounting (shunt)</td>
<td>Hard</td>
<td>Easy</td>
<td>Easy</td>
</tr>
<tr>
<td>Mounting (series)</td>
<td>Easy</td>
<td>Easy</td>
<td>Easy</td>
</tr>
</tbody>
</table>

* Assuming $\varepsilon_r=10$ and $h=0.025$ inch
3-Line Matching Network

• Extend matching network synthesis to multiconductor transmission line (MTL) systems, specifically coupled microstrip

• Characterize MTL systems by their “normal mode parameters”
  – Mode configurations
  – Propagation constants
  – Characteristic impedance matrices
Longitudinal Immittance Matrix Functions (LIMFs)

- Given terminations, the immittance matrices are expressed as “input” longitudinal functions looking toward the load
Calibration Methods

1st order calibration: 6 distinct TRL calibrations were required to deembed the fan-in and fan-out sections

2nd order calibration: 1 multimode TRL calibration required to deembed all fan-ins and fan-outs

Multiconductor transmission line matching network device under test
Bias Networks and Multiline/Multimode TRL

- Self-biasing for DC power done using air bridges
- Regulator circuit may be built on board with surface mount/chip components
- Calibration must include all modes and allow for the 6-port discontinuity S-parameter measurement

Proposed calibration fixture for transistor amplifier device discontinuity with self-biasing network
3-line Transistor Amplifier

The general amplifier structure with generic matching networks

Note that a transistor seating hole may be present for convenience, but is not necessary

Photograph of a three-line transistor amplifier autobaised w/o matching networks
Transistor Amplifier Matching

S-parameter comparison between unmatched and matched three-line amplifier. Dotted line - unmatched; dashed line - matched. Design frequency: 3.1 GHz.

Matching
• 2 - 0.8 pF chip cap.
• 2 - 0.4 pF chip cap.

Gains
• 3.38 dB @ 3.1 GHz
  m=5.14 / um= 1.76
• 6.22 dB @ 3.22 GHz
  m=6.54 / um= 0.32
V Transmission Line

dielectric

ground reference

signal strip

w

h

α

dielectric

ground

2p
V-Line Characteristic Impedance

Calculated values of the characteristic impedance for a single-line v-strip structure as a function of width-to-height ratio w/h. The relative dielectric constant is \( \varepsilon_r = 2.55 \).
Calculated values of the effective relative dielectric constant for a single-line v-strip structure as a function of width-to-height ratio w/h.

The relative dielectric constant is $\varepsilon_r = 2.55$
Three-Line – V Transmission Line
V-Line and Microstrip

**Microstrip**

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (pF/m) =</td>
<td>52.49</td>
<td>-5.90</td>
<td>-0.57</td>
</tr>
<tr>
<td>L (nH/m) =</td>
<td>609.74</td>
<td>113.46</td>
<td>41.79</td>
</tr>
</tbody>
</table>

**V-line**

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (pF/m) =</td>
<td>74.00</td>
<td>-0.97</td>
<td>-0.23</td>
</tr>
<tr>
<td>L (nH/m) =</td>
<td>425.84</td>
<td>20.36</td>
<td>7.00</td>
</tr>
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</table>

Comparison of the inductance and capacitance matrices between a three-line v-line and microstrip structures. The parameters are p/h = 0.8 mils, w/h = 0.6 and $\varepsilon_r = 4.0$. 
Plot of mutual inductance (top) and mutual capacitance (bottom) versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are w/h = 0.24, εr = 4.0.
V-Line vs Microstrip: Coupling Coefficients

Plot of the coupling coefficient versus spacing-to-height ratio for v-line and microstrip configurations. The parameters are w/h = 0.24, εr = 4.0.
Advantages of V-Line

* Higher bandwidth
* Lower crosstalk
* Better transition
V-Line vs Microstrip: Insertion Loss

Insertion Loss

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>V-line</th>
<th>Microstrip</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>10</td>
<td>-2</td>
<td>-2</td>
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<tr>
<td>15</td>
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<td>-3</td>
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<tr>
<td>20</td>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>25</td>
<td>-5</td>
<td>-5</td>
</tr>
</tbody>
</table>

20*log10|S21|