1. Given information: Reflection coefficient is zero and in reference to Figure 1, medium 1 \(\rightarrow (2.25\varepsilon_0, \mu_0)\), medium 2 \(\rightarrow (\varepsilon_0, \mu_0)\).

![Figure 1](image.png)

**Figure 1:**

a) Since \(\varepsilon_1 \neq \varepsilon_2\) which implies that \(\eta_1 \neq \eta_2\) and \(\Gamma = 0\), then the incident wave is a TM wave or \(\parallel\) polarization. With \(\Gamma_{\parallel} = 0\), the incident angle is at the Brewster’s angle.

b) Again, since \(\Gamma_{\parallel} = 0\) then \(\theta_1 = \theta_p\), the incident angle that allows perfect transmission. Thus,

\[
\theta_p = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) = \tan^{-1}\left(\sqrt{\frac{1}{2.25}}\right) = 33.69^\circ
\]

c) To determine the angle of transmission, use the special relationship between the Brewster’s angle and the transmission angle, i.e.,

\[
\theta_p + \theta_2 = 90^\circ \\
\theta_2 = 90^\circ - \theta_p \\
\theta_2 = 56.31^\circ
\]

d) Since \(|\mathbf{H}_i| = 0.1 \text{ A/m}\), the fields in their general TM form can be written as follows. \(\mathbf{H}_i = \hat{y}H_0e^{-jkx}\).

Then with \(\theta_1 = \theta_p\) known,

\[
\mathbf{H}_i = \hat{y}H_0e^{-jk_1(x\cos\theta_p+z\sin\theta_p)} = \hat{y}0.1e^{-j2\pi \frac{2\times 150 \times 10^6 \times 2.25}{3 \times 10^8}(x\cos33.69^\circ+z\sin33.69^\circ)} = \hat{y}0.1e^{-j\frac{3\pi}{2}(x\cos33.69^\circ+z\sin33.69^\circ)} = \hat{y}0.1e^{-j(3.9209x+2.6140z)} \text{ A/m}
\]
- $\mathbf{\tilde{H}}_t = \hat{y}H_0 Te^{-jkz}r$, where $T = (1 + R) \frac{\eta_2}{\eta_1} r$

For $R = 0$, $T = 1$, 
\[
\mathbf{\tilde{H}}_t = \hat{y}H_0 T e^{-jkz(x \cos \theta_2 + z \sin \theta_2)} \\
= \hat{y}0.1e^{-j(2 \pi (x \cos 56.31^o + z \sin 56.31^o))} \\
= \hat{y}0.1e^{-j(1.7426x + 2.6130z)} A/m
\]

(e) For the incident wave, the time-average Poynting vector is 
\[
< \mathbf{\mathbf{\tilde{S}}}_i > = \frac{H_0^2 \eta_1}{2} \hat{k}_i = \frac{0.12120\pi}{2^2} (x \cos 33.69^o + z \sin 33.69^o) W/m^2.
\]

Therefore, its x-component is 
\[
< S_{ix} > = \frac{0.12120\pi}{2^2} \cos 33.69^o = 1.0456 W/m^2.
\]

For the transmitted wave, the time-average Poynting vector is 
\[
< \mathbf{\mathbf{\tilde{S}}}_t > = \frac{H_0^2 \eta_2}{2} \hat{k}_t = \frac{0.12120\pi}{2^2} (x \cos 56.31^o + z \sin 56.31^o) W/m^2.
\]

Therefore, its x-component is 
\[
< S_{tx} > = \frac{0.12120\pi}{2^2} \cos 56.31^o = 1.0456 W/m^2.
\]

We can see that they have the same value for x-components. Therefore, the power is conserved in the x direction.

2. Consider Figure 2 as it relates to the TEM RHCP wave: $\mathbf{\tilde{E}}_i(z) = (\hat{x} - j\hat{y}) e^{-jkz}$, a composition of two linearly polarized waves with the $\hat{y}$ component lagging behind the $\hat{x}$ component by 90° in phase.

a) Apply the boundary condition on the air-PEC interface, $z = 0$, let $\hat{z}$ be normal to the surface and set the tangential components of the total field to zero, i.e., 
\[
\hat{x} \cdot (\mathbf{\tilde{E}}_i(z = 0) + \mathbf{\tilde{E}}_r(z = 0)) \quad \text{and} \quad \hat{y} \cdot (\mathbf{\tilde{E}}_i(z = 0) + \mathbf{\tilde{E}}_r(z = 0)).
\]

The result implies that 
\[
E_x^r = -E_x^i = -1
\]
and 
\[
E_y^r = -E_y^i = +j
\]

Since the reflected wave travels in the $-\hat{z}$ direction, it can easily be written as 
\[
\mathbf{\tilde{E}}_r(z) = (-\hat{x} + j\hat{y}) e^{jkz} \quad (1)
\]
b) It is better served to go to the time domain:

\[
E_i(z, t) = \hat{x} \cos(\omega t - kz) + \hat{y} \cos(\omega t - kz - \pi/2) \\
= \hat{x} \cos(\omega t - kz) - \hat{y} \sin(\omega t - kz)
\]

\[
E_r(z, t) = -\hat{x} \cos(\omega t + kz) + \hat{y} \cos(\omega t + kz + \pi/2) \\
= -\hat{x} \cos(\omega t + kz) - \hat{y} \sin(\omega t + kz) \\
= - (\hat{x} \cos(\omega t + kz) + \hat{y} \sin(\omega t + kz)) .
\]

At \( z = 0 \), the incident and reflected fields differ by a minus sign and thus the field vectors rotate in the same direction as a function of time.

As a physical exercise to verify the result do the following: Take your right hand and your left hand, point your thumbs toward each other, curl your fingers over and over again—the curling fingers represent the field vectors and thus rotate in the same direction. The right hand represents the RHCP wave and the left hand represents the LHCP wave, the thumbs point in the direction of propagation for each.

c) One approach is to look at the problem graphically in the time domain, i.e., examining the locus of the instantaneous reflected electric field vector at the \( z = 0 \) plane for some sample times. Consider the plot in Figure 3 which shows the locus of the reflected electric field vector in (2) for various times in the \( z = 0 \) plane.

From the figure, one can see as \( \omega t \) starts from 0 and increases, the electric field vector rotates in the clockwise direction. Recall the definitions for RHCP and LHCP:

- RHCP: The electric field has a clockwise sense of direction when it is viewed along the axis of propagation
- LHCP: The electric field has a counterclockwise sense of direction when it is viewed along the axis of propagation
Since the rotation is clockwise, you might think RHCP...not correct because you are “in front” of the wave, not “behind”. If you can imagine yourself on the other side of the page, behind the wave—it is propagating away from you—you would discern a counterclockwise rotation and conclude the wave is LHCP.

- One can also determine the same results by using the phasor approach in relation to (1) in the \(x-y\) plane (reference Figure 3). That is, curl the phase leading component (\(\hat{y}\)) into the phase lagging component (\(-\hat{x}\))—the rotation direction is again counterclockwise when viewed from behind the wave. Therefore the wave is LHCP.

3. The problem geometry can be modeled via image theory as in the Figure 4. Note the direction of the image currents (phases) with respect to the physical short dipole. Image theory allows us to model and solve what would rather be a more complicated calculation into a much simpler one. The direction of the image currents are chosen such that boundary conditions (field representation) equates to the original problem. When the original and image currents are configured as in Figure 4, array theory can be used to determine the far field solution.

You might have recognized this type of radiation structure in base-station and/or cell phone towers with the antenna covered by a radome (weather proofer). The idea is to encapsulate the radiating element inside the metallic casing for a more directive pattern within a ±45°sector.

a) It is desired to design the corner reflector such that maximum radiation direction (the field strength of the antenna is four times that of the single dipole) occurs along the \(+x (\phi = 0^\circ)\) direction in the far field.

Due to the problem geometry, utilize pattern multiplication to determine the total far field (i.e., the total pattern of an array of identical elements can be interpreted as the element factor multiplied by the array factor).

\[
\mathbf{E}_{\text{total}}(\mathbf{r}) = \mathbf{E}_{\text{ref}}(\mathbf{r}) \times AF,
\]
where $\tilde{E}_{\text{ref}}(\mathbf{r})$ is the field of the single short dipole located at $(d, 0, 0)$. Note, the solution is valid only within $\pm 45^\circ$.

$$
\left| \frac{\tilde{E}_{\text{total}}(\mathbf{r})}{\tilde{E}_{\text{ref}}(\mathbf{r})} \right| = AF = 4
$$

then

$$
AF = I_0 e^{jkd \cos \phi} + I_0 e^{jkd \sin \phi} + I_0 e^{jkd \cos \phi} - I_0 e^{jkd \sin \phi} + I_0 e^{jkd \cos \phi} - I_0 e^{jkd \sin \phi} + I_0 e^{jkd \cos \phi} - I_0 e^{jkd \sin \phi}
$$

The current amplitude $I_0$ becomes an external scaling factor and thus

$$
AF = 2 \left[ \cos (kd \cos \phi) - \cos (kd \sin \phi) \right].
$$

When does the magnitude in (3) equal to 4? With the observation point being at $\phi = 0^\circ$, the evaluation is easy.

$$
|AF| = |2 \left[ \cos (kd \cos \phi) - \cos (kd \sin \phi) \right]| = 4
$$

$$
2 \left[ \cos (kd \cos \phi) - \cos (kd \sin \phi) \right] = \pm 4
$$

$$
2 \left[ \cos (kd \cos 0) - \cos (kd \sin 0) \right] = \pm 4
$$

$$
(cos kd - 1) = \pm 2
$$

$$
\cos kd = \{3, -1\}.
$$
Hence,
\[
\begin{align*}
\cos kd &= -1 \\
kd &= n\pi \quad n \in \text{integer} \\
\Rightarrow \quad d &= \frac{\lambda}{2} \quad (\text{the smallest possible value})
\end{align*}
\]

b) The radiation field strength can be determined from the Total Pattern expressed above. Since we know for a \( z \)-directed short dipole,
\[
\tilde{E}_{ref}(r) = j\eta_0 I_0 k_0 \frac{L}{2} e^{-jkr} \sin \theta \hat{\theta},
\]
then the total field can be expressed as
\[
\tilde{E}_{total}(r) = j\eta_0 I_0 k_0 \frac{L}{2} \frac{e^{-jkr}}{4\pi r} \sin \theta \hat{\theta} \times 2 (\cos (kd \cos \phi) - \cos (kd \sin \phi)).
\]
Then along the \( \hat{x} \) direction (\( \theta = 90^\circ, \phi = 0^\circ \)),
\[
\tilde{E}_{total}(r) = j\eta_0 I_0 k_0 \frac{L}{2} \frac{e^{-jkr}}{4\pi x} (-\hat{z}) \times 4.
\]
The strength of the field is essentially the magnitude and thus
\[
\left| \tilde{E}_{total}(x = 1\text{km}) \right| = 4 \times 120\pi \times 1 \frac{2\pi \lambda/10}{\lambda/2} \frac{1}{4\pi \times 1000} = \frac{3\pi}{250} \text{ V/m} = 0.03770 \text{ V/m}
\]

4.

a) See the plot below, in which the image of the dipole antenna is in the position to make sure that the tangential component of the electrical field on the perfectly conducting surface is zero.

b) The A.F. is
\[
AF = \left| \frac{\tilde{E}_{total}(r)}{\tilde{E}_{ref}(r)} \right| = e^{jk\hat{r} \cdot r'} + \frac{I_{01}^2}{I_{01}} e^{jk\hat{r} \cdot r'} = 1 + 0 e^{-j2d\sin 90^\circ \cos \phi} = 1 - e^{-j2d\sin 0^\circ \cos \phi} = 1 - e^{-j3\pi \cos \phi}.
\]

c) \((x, y, z) = (10\text{km}, 0, 0),\) we have \( \phi = 0^\circ, \)
\[
AF = 1 - e^{-j3\pi} = 2.
\]
d) Since \( AF = 2 \), so

\[
\tilde{E}_{\text{ref}}(r) = \frac{\tilde{E}_{\text{total}}(r)}{2} = -\hat{z} 5 \text{ V/m}.
\]

5. Observe the fine details in the problem statement:

- The electric field phasor at the origin is given as

\[
\tilde{E}_i(0, 0, 0) = -\frac{\sqrt{3}}{2} \hat{x} + j \hat{y} + \frac{1}{2} \hat{z}. \tag{4}
\]

From the standard plan wave form \( \tilde{E} = \hat{e}_r E_0 e^{-j\mathbf{k} \cdot \mathbf{r}} \), evaluation of the field at the origin gives the polarization and amplitude of the field. Rewriting (4) as

\[
\tilde{E}_i(0, 0, 0) = \left( -\frac{\sqrt{3}}{2} \hat{x} + \frac{1}{2} \hat{z} \right) + j \hat{y}
\]

clearly shows that the electric field is a linear combination of a TE and TM wave. Therefore all the governing relationships learned for each mode apply.

- The problem states that the reflected field is linearly polarized, despite the fact that the incident wave is circular polarized (CP). Instead of the reflected wave being CP as well, something special happens—that being the TM component of the of the incident wave is fully transmitted \( (\Gamma_{\parallel} = 0) \) and thus is incident at the Brewster’s angle. It has to be the TM component because the reflection coefficient cannot be zero in the TE case when \( \mu_1 = \mu_2 \) and \( \epsilon_1 \neq \epsilon_2 \).

a) To find the angle of incidence w.r.t. the normal in the \( x - z \) plane, use the geometry and what the plane wave forms of Maxwell’s Equations dictate: \( \mathbf{E}, \mathbf{H} \), and \( \hat{k} \) form a triad and are perpendicular to each other. See Figure 6. The incident electric field direction in terms of its polarization components are drawn at the origin and a projection of the field in space which is incident on the interface. Note the polarization of the incident field does not change. (Why can’t we choose \( \hat{k} \) in the \( y \) direction?)
Since \( \hat{e} \) is a unit vector, then from the geometry displayed in Figure 6

\[
\cos \theta = \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin \theta = \frac{1}{2} \implies \theta = 30^\circ
\]

\[
\theta + 90 + \theta_i = 180 \implies \theta_i = 60^\circ
\]

b) As previously discussed, the incident field is impinging at the Brewster’s angle and thus one can solve directly in terms of the incident angle. Given that \( \theta_p = \theta_i \), then

\[
\tan \theta_p = \sqrt{\frac{\epsilon_2}{\epsilon_1}}
\]

\[
\epsilon_{r2} = \epsilon_{r1} \tan^2 \theta_p
\]

\[
\epsilon_{r2} = 1 \tan^2 60^\circ = 3
\]

c) Use the special relationship between the Brewster’s angle and the transmission angle:

\[
\theta_p + \theta_t = 90^\circ
\]

\[
\theta_t = 90^\circ - \theta_p
\]

\[
\theta_2 = 30^\circ
\]
d) Given the incident and transmission angles and the medium properties,

\[ k_i = k_1 = \frac{2\pi}{\lambda_1} (\hat{x} \cos \theta_1 + \hat{z} \sin \theta_1) \]

\[ = \frac{2\pi}{10^{-6}} (\hat{x} \cos 60^\circ + \hat{z} \sin 60^\circ) \]

\[ = 2\pi 10^6 \left( \frac{1}{2} \hat{x} + \frac{\sqrt{3}}{2} \hat{z} \right) \]

\[ = \pi 10^6 \left( \hat{x} + \sqrt{3} \hat{z} \right) \text{ rad/m} \]

and

\[ k_t = k_2 = \frac{2\pi}{\lambda_2} (\hat{x} \cos \theta_2 + \hat{z} \sin \theta_2) \]

\[ = \frac{2\pi}{10^{-6}} \sqrt{3} (\hat{x} \cos 30^\circ + \hat{z} \sin 30^\circ) \]

\[ = 2\pi 10^6 \sqrt{3} \left( \frac{\sqrt{3}}{2} \hat{x} + \frac{1}{2} \hat{z} \right) \]

\[ = \pi 10^6 \left( 3\hat{x} + \sqrt{3} \hat{z} \right) \text{ rad/m} \]

e) It’s been determined that only the TM component of the incident electric field is fully transmitted, i.e., \( R = \Gamma_\parallel = 0 \) and thus \( T = 1 \) implies \( \gamma_\parallel = \frac{\eta_2}{\eta_1} \). Also it is known that the reflected field is linearly polarized, the TE mode. This does not mean that the entire TE mode is reflected (lectures on total internal reflection are to come), instead part of it is reflected and part of it is transmitted. Determine \( \Gamma_\perp \) to fully express the reflected field.

\[ \Gamma_\perp = \frac{E_{yr}}{E_{yi}} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cos \theta_1 - \frac{\eta_0}{\sqrt{\epsilon_r}} \cos \theta_2 \]

\[ \Gamma_\perp = \frac{1}{\sqrt{3}} \cos 60^\circ - \frac{\eta_0}{\sqrt{\epsilon_r}} \cos 60^\circ \]

\[ = -\frac{1}{2} \]

Since \( E_{yr} = \Gamma_\perp E_{yi} = -\frac{1}{2} E_{yi} \), the reflected phasor can be written as

\[ \vec{E}_r = \hat{y} \Gamma_\perp E_{yi} e^{-jk_r \cdot r} \]

\[ = \hat{y} \left( -\frac{1}{2} e^{-j\pi 10^6 (\sqrt{3} \hat{x})} \right) \]

\[ = -\hat{y} \frac{j}{2} e^{-j10^6 (-3.1412\hat{x} + 5.441\hat{z})} \text{ V/m} \]
f) To find the transmitted field, determine $\tau_\perp$ to find out how much of the incident TE mode is transmitted into the dielectric medium.

$$\tau_\perp = \frac{E_{gt}}{E_{yi}} = 1 + \Gamma_\perp$$

then

$$E_{yt} = \tau_\perp E_{yi} = (1 + \Gamma_\perp) E_{yi}$$

and since $\tau_\perp = \frac{1}{2}$

$$E_{yt} = \frac{1}{2} E_{yi}.$$ 

For the TM mode, recall, as given in the lecture notes

$$T = \eta_1 \frac{E_t}{E_i} = \eta_1 \frac{\tau_\parallel}{\eta_2}.$$ 

Therefore, with $T = 1$, $E_t = \frac{\eta_2}{\eta_1} E_i$, where $E_i$ and $E_t$ refer to the amplitudes of the incident and transmitted TM field vectors at the origin pointing in reference directions along $-\hat{k} \times \hat{H}$. See Lecture note 17, page 3 for a graphical representation of the directions.

Again, given the fact that $\Gamma_\parallel = 0$ and $\tau_\parallel = \frac{\eta_2}{\eta_1} = \frac{1}{\sqrt{3}}$, one can write the transmitted field phasor as

$$\tilde{E}_t = (\tau_\perp E_{yi} + \tau_\parallel E_t) e^{-jk_2 \cdot r}$$

$$= \left( \frac{\hat{y}}{2} + \tau_\parallel \left( -\hat{k}_2 \times \hat{h}_2 \right) \right) e^{-j\pi 10^6 (3x + \sqrt{3}z)}$$

$$= \left( \frac{\hat{y}}{2} + \tau_\parallel \left[ - \left( \frac{\sqrt{3}}{2} \hat{x} + \frac{1}{2} \hat{z} \right) \times -\hat{y} \right]\right) e^{-j\pi 10^6 (-3x + \sqrt{3}z)}$$

$$= \left( \frac{\hat{y}}{2} + \frac{1}{\sqrt{3}} \left( -\hat{x} + \frac{\sqrt{3}}{2} \hat{z} \right) \right) e^{-j\pi 10^6 (3x + \sqrt{3}z)}$$

$$= \left( -\frac{1}{2\sqrt{3}} \hat{x} + \frac{\hat{y}}{2} + \frac{\sqrt{3}}{2} \hat{z} \right) e^{-j10^6 (9.425x + 5.441z)} V/m.$$ 

The result yields an elliptically polarized wave, no longer does the tip of the electric field vector trace out a circle of constant magnitude.

As an exercise, one should check to make sure boundary conditions holds at the $x = 0$ interface.

6. In medium 1, the glass region,

$$\tilde{E}_i = \frac{\hat{x} - \hat{z}}{\sqrt{2}} e^{-j2\pi \frac{x + z}{\sqrt{2}}} V/m.$$ 

The expression equates to the more general form of a plane wave given by

$$\tilde{E} = \tilde{E}_0 e^{-jk \cdot r}.$$ 

a)
i. Since $k_i = \hat{k}_i k_1$ then from the $e^{-j2\pi \frac{x + z}{\sqrt{2}}}$ term, one may intuitively see that

$$\hat{k}_i = \frac{\hat{x} + \hat{z}}{\sqrt{2}},$$

of which satisfies $\hat{k}_i \cdot \hat{k}_i = 1$ and thus $k_1 = 2\pi$.

Another approach is to equate $e^{-j2\pi \frac{x + z}{\sqrt{2}}}$ to $e^{-j(k_{x1}x + k_{z1}z)}$. Do so yields

$$k_{x1} = \frac{2\pi}{\sqrt{2}} \text{ and } k_{z1} = \frac{2\pi}{\sqrt{2}}$$

which implies that

$$k_1 = \sqrt{k_{x1}^2 + k_{z1}^2} = 2\pi$$

ii. $k_1 = \frac{2\pi}{\lambda_1} = \frac{\omega}{v_p}$ then

$$\omega = k_1 v_p = \frac{k_1 c}{\sqrt{\varepsilon_r}} = \frac{2\pi \cdot 3 \times 10^8}{\sqrt{2.25}} = 1.26 \times 10^9 \text{ rad/m}$$

b) i. Given $\hat{e}$ from the incident field, determine $\theta_i$ graphically knowing that (1) the incident angle is in the plane of incidence and (2) $\hat{E}_i$ is $\perp$ to the direction of propagation. As a matter of fact, since $\hat{k}_i$ is discernible from the given incident fields, determining $\theta_i$ is much easier. In reference to Figure 7, it is easy to see that

$$\hat{k}_{ix} = \cos \theta_i = \frac{1}{\sqrt{2}}$$

and

$$\hat{k}_{iz} = \sin \theta_i = \frac{1}{\sqrt{2}},$$

which implies that $\theta_i = 45^\circ$.

ii. To find $\theta_c$ use Snell’s Law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2,$$

then with $\theta_2 = 90^\circ$

$$\sin \theta_c = \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_r_2}{\varepsilon_r_1}} = \sqrt{\frac{1}{2.25}} = \frac{2}{3}$$

$$\therefore \theta_c = \sin^{-1} \frac{1}{2.25} = 41.8^\circ$$
An evanescent wave will be produced in the half space $x > 0$ because the angle of incidence is greater than the critical angle. That is, $\theta_i = 45^\circ > \theta_c = 41.8^\circ$.

This scenario leads to total internal reflection (TIR) of the incident field impinging upon the interface.

c)

i. Starting from Snell’s law,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1,$$

since

$$\cos \theta_2 = \sqrt{1 - \sin^2 \theta_2}$$

then

$$\cos \theta_2 = \sqrt{1 - \frac{n_1^2}{n_2^2} \sin^2 \theta_1}.$$ 

In TIR conditions, $\frac{n_1^2}{n_2^2} \sin^2 \theta_1 > 1$, thus

$$\cos \theta_2 = \sqrt{-1 \left( \frac{n_1^2}{n_2^2} \sin^2 \theta_1 - 1 \right)}$$

$$= \pm j \sqrt{\frac{n_1^2}{n_2^2} \sin^2 \theta_1 - 1}$$

$$= \pm j \alpha$$
\[ \cos \theta_2 = \pm j \sqrt{\left(\frac{3}{2}\right)^2 \sin^2 45^\circ - 1} = \pm j 0.354 \]
\[ \rightarrow \cos \theta_2 = -j 0.354 \]

ii.

- \( \theta_2 \) is complex because \( n_1 > n_2 \) and \( \theta_i > \theta_c \). As the wave travels from medium 1 to medium 2, TIR occurs and thus the complex transmission angle indicates the emergence of an evanescent wave traveling along the glass-air interface.

- Choose \( \cos \theta_2 = -j 0.354 \) and not \( \cos \theta_2 = +j 0.354 \). Why? Recall, the \( \cos \theta_2 \) term is included in the transmission term, \( k_t \), i.e., \( \tilde{E}_i \propto e^{-jk_2(\cos \theta_2 x + \sin \theta_2 z)} \). Since it is complex, it makes the \( x \)-dependent factor of the “phase” term equivalent to \(-jk_2(\pm j0.354)x = \pm k_2 0.354x\). This result is no longer complex, it becomes a field amplitude factor via the exponential.

Thus one must choose the \(-j0.354\) term because choosing the \(+j0.354\) term means the amplitude of transmitted field grows without bound as \( x \) increases (and decays as \( x \) decreases), which means that at the source the field would approach zero—does not make sense). This violates a number of physical laws including the radiation conditions which states that fields must decay as they propagate away from the source—a consequence due to conservation of energy. Fields cannot grow as distance increases, where would the energy coming from! Recall the spherical waves radiated from the linear dipoles—they went as \( 1/r \) signifying conservation of energy due to spherical spreading. The same concept applies here. The \(-j0.354\) term satisfies all the conditions just mentioned.

\[ \frac{H_{yt}}{H_{yi}} = \frac{\eta_1}{\eta_2} \tau \] is the same as finding \( T \), the transmission coefficient for magnetic
fields. One approach would be to utilize

\[ H_{yi} = \frac{\eta_1}{\eta_2} \tau || = \frac{2\eta_1 \cos \theta_1}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2} \]

or the simpler way would be use \( T = 1 + R \).

\[ T = 1 + R = 1 - \Gamma || = 1.278 + j0.9603 = 1.599e^{-j0.6441(\text{rad})} = 1.599 \angle 36.9^\circ \]

f) In this case, it is easier to work with \( H \) since it is only \( y \)-polarized.

\[ \tilde{H}_i = \frac{k_i \times \tilde{E}_i}{\eta_1} = \frac{1}{\eta_1} \frac{\hat{x} + \hat{z}}{\sqrt{2}} \times \frac{\hat{x} - \hat{z}}{\sqrt{2}} e^{-j2\pi \frac{x+z}{\sqrt{2}}} = \frac{\sqrt{\epsilon_r 1}}{\eta_0} \left( \frac{1}{2} \hat{y} + \frac{1}{2} \hat{y} \right) e^{-j2\pi \frac{x+z}{\sqrt{2}}} \]

\[ = 0.00398 \hat{y} e^{-j2\pi \frac{x+z}{\sqrt{2}}} \]

Then using \( \frac{H_{yi}}{H_{yi}} \),

\[ \tilde{H}_t = \hat{y} T H_{yi} e^{-jk_2(\cos \theta_2 x + \sin \theta_2 z)} \]

\[ = \frac{2\eta_1 \cos \theta_1}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2} H_{yi} e^{-jk_2(\cos \theta_2 x + \sin \theta_2 z)} \]

\[ = \frac{2\eta_1 \cos \theta_1}{\eta_1 \cos \theta_1 - \eta_2 j \sqrt{\frac{n_1^2}{n_2^2} \sin^2 \theta_1 - 1}} H_{yi} \left[ -j k_2 \left( \frac{n_2^2}{n_1^2} \sin^2 \theta_1 - 1 \right) e^{-j(\frac{n_2^2}{n_1^2} \sin^2 \theta_1 - 1) x} + \frac{k_1}{k_2} \sin \theta_1 \right] \]

\[ = \frac{\hat{y}}{\eta_1} \left( 1.599e^{j0.6441(\text{rad})} \right)^{3/2} e^{-j \frac{\omega}{c} \left[ -j0.354x + (3/2 \sin 45^\circ)z \right]} \]

\[ = \frac{\hat{y}}{\eta_1} \left( 1.599e^{j0.6441(\text{rad})} \right)^{0.00398} e^{-j 4.189 \left[ -j0.354x + 1.061z \right]} \]

\[ = \frac{\hat{y}}{\eta_1} 6.364 \times 10^{-3} e^{j0.6441} \left( e^{-1.483x} e^{-j 4.44z} \right) H/m \]
ii. Next obtain $\tilde{E}_t$ from $\tilde{H}_t$,

$$\tilde{E}_t = \frac{\eta_2}{\kappa_2} \tilde{H}_t \times \hat{k}_t$$

$$= 120\pi \left[ \hat{y} \times 6.364 \times 10^{-3} e^{j0.6441} \left( e^{-1.483x} e^{-j4.44z} \right) \right] \times \left[ \left( -j \sqrt{\frac{n_1^2}{n_2^2}} \sin^2 \theta_1 - 1 \right) \hat{x} + \left( \frac{\kappa_1}{\kappa_2} \sin \theta_1 \right) \hat{z} \right]$$

$$= 2.399 e^{j0.6441} \left( e^{-1.483x} e^{-j4.44z} \right) \hat{y} \times \left( -j 0.354 \hat{x} + 1.061 \hat{z} \right)$$

$$= 2.399 e^{j0.6441} \left( e^{-1.483x} e^{-j4.44z} \right) \left( \hat{z} 0.354 \hat{x} + \hat{x} 1.061 \right)$$

$$= (\hat{x} 1.061 + \hat{z} 0.354) \times 2.399 e^{j0.6441} \left( e^{-1.483x} e^{-j4.44z} \right) \frac{V}{m}$$

\[\frac{1}{\eta_2} |\tilde{E}_t|^2 \hat{k}_t \quad \text{STOP!!} \quad \text{It is incorrect to use this expression here. Recall, it is a simplified version of the time-average Poynting vector for plane waves. No longer does the wave propagating in air resemble a plane wave, it is a non-uniform evanescent wave. Utilize the general expression}\]

$$\langle S_t \rangle = \frac{1}{\eta_2} \left| \tilde{E}_t \right|^2 \hat{k}_t$$

As expected, real power flow is along the interface, attenuating away from it in the $+x$ direction.