1. A space ship traveling from Earth to Moon with some velocity \( v \) (relative to Earth) is emitting a TEM wave at a radian frequency \( \omega \). The TEM wave reaching Earth is found to be oscillating with a radian frequency of \( \omega_E = 2.99 \times 10^9 \) rad/sec while on the moon the wave frequency is measured as \( \omega_M = 3.01 \times 10^9 \) rad/sec.

a) Determine \( \omega, k \) and \( \lambda \), where \( k \) and \( \lambda \) are the TEM wavenumber and wavelength, respectively, in the reference frame of the space ship. Assume free-space propagation and that the distance between Earth and Moon is constant during the measurements.

b) Determine \( v \). Is this a realistic velocity for a space ship? For comparison’s sake, light travels from Earth to Moon in about 1.25 s.

2. An electron is moving at a large speed \( v \) in \( x \)-direction towards a \( z \)-polarized dipole located at the origin. In response to the radiation coming from the dipole at a frequency \( \omega \), the electron oscillates at a Doppler-shifted frequency \( \omega' \), and radiates a wave that is detected in the reference frame of the dipole at a frequency \( \omega'' = \frac{5}{3} \omega \).

a) What polarization do you expect for the field coming from the oscillating electron? Briefly explain your reason.

b) What is the electron speed \( v \)? Justify your answer by explaining the reasons for which Doppler formula you used.

c) What is the frequency \( \omega' \)?

3. Consider a police radar operating at a frequency of \( f = 1 \) GHz. Let \( f'' \) denote the frequency of the reflected signal from your car back to the antenna of the police radar when you are moving with a speed of 80 mph on the highway.

a) What is the maximum possible value of \( f'' - f \)? Calculate this quantity using both the relativistic and non-relativistic Doppler shift formulae. Any perceptible difference between them?

b) What is the minimum possible \( |f'' - f| \)? Explain in terms of geometry and the rate of change of phase delay of the radar signal going from the radar antenna to your car and then from your car back to the radar antenna — remember: phase delay depends on the distance traveled by the wave!

4. Consider a “mothership” which launches a probe into the atmosphere of a planet. The probe descends into the atmosphere at a speed of 75 km/s relative to the mothership. The probe transmits a signal at a frequency of 2.3 GHz.

a) At what frequency should the mothership expect to receive the probe’s signal on?

b) After the probe lands, the mothership leaves the planet at a speed of 100,000 km/s relative to the probe. At what frequency should the mothership now expect to receive the probe’s signal on?